Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $Grad(1/r) = \frac{-\vec{r}}{r^3}$.

2. Evaluate $\int_C (yz\vec{i} + xz\vec{j} + xy\vec{k}) \cdot d\vec{r}$ where C is the boundary of a surface S.

3. Solve $(D^3 - 3D^2 + 3D - 1)y = 0$.

4. Obtain the differential equation of x alone, given $x' = 7x - y$ and $y' = 3x + y$.

5. Prove that $L\left(\int_0^t f(t)\,dt\right) = \frac{F(s)}{s}$, where $L(f(t)) = F(s)$.

6. Find $L^{-1}\left(\log \frac{s}{s-a}\right)$.

7. Prove that the family of curves $u = c$, $v = k$ cuts orthogonally for an analytic function $f(z) = u + iv$.

8. Find the invariant points of a function $f(z) = \frac{z^3 + 7z}{7 - 6zi}$.

9. Define and give an example of essential singular points.

10. Express $\int_0^{\pi} \frac{d\theta}{2\cos\theta + \sin\theta}$ as complex integration.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the values of constants a,b,c so that the maximum value of the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1,2,-1)$ has a magnitude 64 in the direction parallel to z-axis. (6)

(ii) Verify Green's theorem for $\vec{F} = (x^2+y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$. (10)

Or

(b) (i) A fluid motion is given by $\vec{V} = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$. Is this motion irrotational and is this possible for an incompressible fluid? (6)

(ii) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$. And S is the surface of the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0$ and $z = c$. (10)

12. (a) (i) Solve $(D^3 + 2D^2 + D)y = e^{-x} + \cos 2x$. (8)

(ii) Solve $y'' = x, x'' = y$. (8)

Or

(b) (i) Solve $y'' + y = \sec x$. (6)

(ii) Solve $(2x+7)^2 y'' - 6(2x+7)y' + 8y = 8x$. (10)

13. (a) (i) Find $L(e^{-t}\sin^2 3t)$ and $L\left(\frac{e^{-t} - \cos t}{t}\right)$. (3 + 3)

(ii) Solve $x'' + 2x' + 5x = e^{-t}\sin t$; $x(0) = 0$ and $x'(0) = 1$ using Laplace transform. (10)

Or

(b) (i) State second shifting theorem and also find $L^{-1}\left(\frac{e^{-s}}{\sqrt{s+1}}\right)$. (2 + 4)

(ii) Find $L^{-1}\left(\frac{3s+1}{(s+1)^4}\right)$. (4)

(iii) Find the Laplace transform for $f(t) = \sin\frac{\pi t}{a}$, such that $f(t+a) = f(t)$. (6)

14. (a) (i) If $u = x^2 - y^2$, $v = \dfrac{y}{x^2 + y^2}$; prove that u and v are harmonic functions but $f(z) = u + iv$ is not an analytic function. (6)

(ii) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is a real part of an analytic function. Also find its conjugate harmonic function v and express $f(z) = u + iv$ as function of z. (10)

Or

(b) (i) Is $f(z) = z^n$ analytic function everywhere? (4)

(ii) Find the image of the lines $u = a$ and $v = b$ in w-plane into z-plane under the transformation $z = \sqrt{w}$. (6)

(iii) Find the bilinear transformation which maps $I, -i, 1$ in z-plane into $0, 1, \infty$ of the w plane respectively. (6)

15. (a) (i) Using Cauchy's integral formula evaluate $\oint_C \dfrac{e^{2z}}{(z+1)^4}\,dz$ where C is $|z| = 2$. (4)

(ii) Evaluate $\int_0^\infty \dfrac{dx}{x^4 + a^4}$ using contour integration. (12)

Or

(b) (i) Obtain the Laurent's expansion of $f(z) = \dfrac{z^2 - 4z + 2}{z^3 - 2z^2 - 5z + 6}$ in $3 < |z+2| < 5$. (6)

(ii) Evaluate $\int_C \dfrac{z^3\,dz}{(z-1)^4(z-2)(z-3)}$ where C is $|z| = 2.5$; using residue theorem. (10)